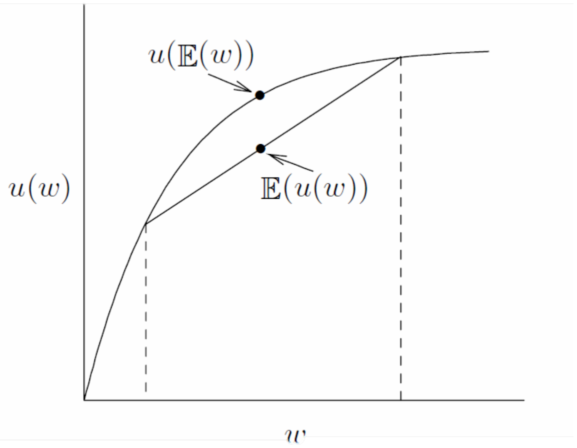
Behavioral economics-Prospect theory

謝秉儒

history

- 很久以前，期望值理論
- 發現聖彼得堡悖論—考慮一個遊戲，不斷地擲同一枚硬幣，直到得到正面為止，如果你擲了X次才最終得到正面，你將獲得2^{X-1}元。遊戲的報名費是100萬元，如果我們考慮到這個遊戲的期望收益是無窮大，我們就應該參加。
- Daniel Bernoulli(1700-1782):在1738年提出預期效用假說(Expected utility hypothesis)，用邊際效用遞減解釋為什麼大家不會去玩這個遊戲。



為什麼預期效用假說會被推翻呢？

- 來玩個小測試（從論文抄下來的）

A: 50% chance to win 1,000, 50% chance to win nothing; B: 450 for sure.

PROBLEM 1: Choose between

A:	2,500 with probability	.33,	B:	2,400 with certainty.
	2,400 with probability	.66,		
	0 with probability	.01;		

PROBLEM 2: Choose between

C:	2,500 with probability	.33,	D:	2,400 with probability	.34,
	0 with probability	.67;		0 with probability	.66.

為什麼預期效用假說會被推翻呢？

- 從1
- u(2,400)> .33u(2,500) + .66u(2,400) 移項 .34u(2,400)> .33u(2,500)
- 但2
- .33u(2,500)> .34u(2,400)
- 負的前景

TABLE I

PREFERENCES BETWEEN POSITIVE AND NEGATIVE PROSPECTS

	Positive prospects				Negative prospects		
Problem 3:	(4,000, .80)	<	(3,000).	Problem 3′:	(−4,000, .80)	>	(−3,000).
$N = 95$	[20]		[80]*	$N = 95$	[92]*		[8]
Problem 4:	(4,000, .20)	>	(3,000, .25).	Problem 4′:	(−4,000, .20)	<	(−3,000, .25).
$N = 95$	[65]*		[35]	$N = 95$	[42]		[58]
Problem 7:	(3,000, .90)	>	(6,000, .45).	Problem 7′:	(−3,000, .90)	<	(−6,000, .45).
$N = 66$	[86]*		[14]	$N = 66$	[8]		[92]*
Problem 8:	(3,000, .002)	<	(6,000, .001).	Problem 8′:	(−3,000, .002)	>	(−6,000, .001).
$N = 66$	[27]		[73]*	$N = 66$	[70]*		[30]

為什麼預期效用假說會被推翻呢？

Problem 9: Suppose you consider the possibility of insuring some property against damage, e.g., fire or theft. After examining the risks and the premium you find that you have no clear preference between the options of purchasing insurance or leaving the property uninsured.

假設有一個保險你思考了一下覺得保這個險不好也不壞（對你效用一樣），接下來保險公司提出了新的方案。你先付一半的保費

當意外發生，他有一半的機會你需要付完剩下一半的錢，並賠償你說好的賠償金。

有另外一半的機會你拿回付過的一半保費，並且不理賠你。

這樣要選這個保險嗎？

為什麼預期效用假說會被推翻呢？

w: 你的asset，p:意外發生機率，y:保費，x:會損失的錢

In contrast to these data, expected utility theory (with a concave u) implies that probabilistic insurance is superior to regular insurance. That is, if at asset position w one is just willing to pay a premium y to insure against a probability p of losing x, then one should definitely be willing to pay a smaller premium ry to reduce the probability of losing x from p to $(1-r)p$, $0<r<1$. Formally, if one is indifferent between $(w-x, p; w, 1-p)$ and $(w-y)$, then one should prefer probabilistic insurance $(w-x, (1-r)p; w-y, rp; w-ry, 1-p)$ over regular insurance $(w-y)$.

To prove this proposition, we show that

$$pu(w-x)+(1-p)u(w)=u(w-y)$$

implies

$$(1-r)pu(w-x)+rpu(w-y)+(1-p)u(w-ry)>u(w-y).$$

Without loss of generality, we can set $u(w-x)=0$ and $u(w)=1$. Hence, $u(w-y)=1-p$, and we wish to show that

$$rp(1-p)+(1-p)u(w-ry)>1-p \qquad \text{or} \qquad u(w-ry)>1-rp$$

which holds if and only if u is concave.

The Isolation Effect

- 還記得problem 3&4嗎？
- Problem3:(4,000,0.8) and (3,000)
- Problem4:(4,000,0.2) and (3,000,0.25).

PROBLEM 10: Consider the following two-stage game. In the first stage, there is a probability of .75 to end the game without winning anything, and a probability of .25 to move into the second stage. If you reach the second stage you have a choice between

(4,000, .80) and (3,000).

決策點不一樣了

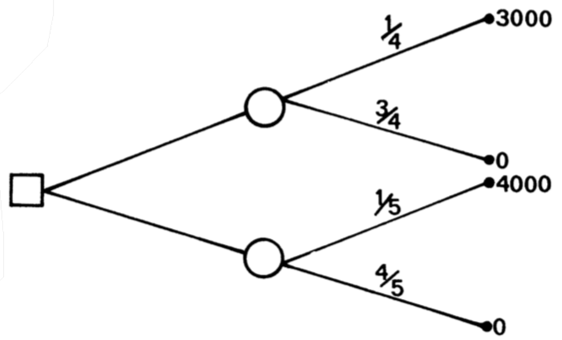


FIGURE 1.—The representation of Problem 4 as a decision tree (standard formulation).

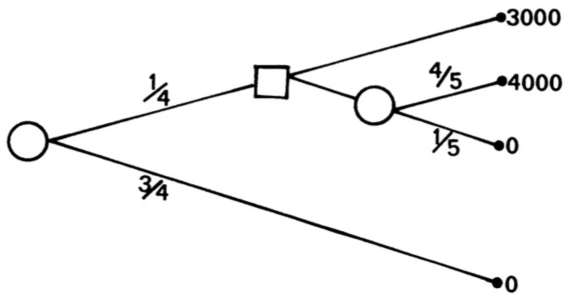


FIGURE 2.—The representation of Problem 10 as a decision tree (sequential formulation).

The Isolation Effect

PROBLEM 11: In addition to whatever you own, you have been given 1,000.
You are now asked to choose between

A: (1,000, .50), and B: (500).

$N = 70$ [16] [84]*

PROBLEM 12: In addition to whatever you own, you have been given 2,000.
You are now asked to choose between

C: (−1,000, .50), and D: (−500).

$N = 68$ [69*] [31]

- 對獲利風險趨避，對損失風險偏好
- 會對根據基準點不同有不一樣的選擇

		績效目標（參考點）	
		a.低標準100萬（獲得の情境）	b.高標準300萬（損失の情境）
1.保守策略（風險趨避）	確定可達成 200萬 業績	+100萬	-100萬
2.冒險策略（風險偏好）	50% 達成 300萬 業績 50% 達成 100萬 業績	50% 0 50% +200萬	50% -200萬 50% 0

reference: http://wfhstudy.blogspot.com/2012/08/blog-post.html

Prospect theory

$$V(x,p;y,q) = \pi(p)v(x) + \pi(q)v(y)$$

- The Weighting Function π(p) ：對機率的function
- The Value Function v(x) ：對財產的function (其實這蠻像Utility的)

If $p+q=1$ and either $x>y>0$ or $x<y<0$, then

(2) $$V(x,p;y,q) = v(y) + \pi(p)[v(x) - v(y)].$$

- 有點類似這樣就把v(y)看作基準點，去展望這件事的機率期望值
- 這是整合理論，但其實有可能失敗。
- The location of the reference point, and the manner in which's choice problems are coded and edited emerge as critical factors in the analysis of decisions.

Value function

- An essential feature of the present theory is that the carriers of value are changes in wealth or welfare, rather than final states.
- ” Many sensory and perceptual dimensions share the property that the psychological response is a concave function of the magnitude of physical change.”
- “許多感官和知覺維度都具有心理反應是身體變化幅度的凹函數的特性。”
- 恰變差！100塊錢跟200塊錢>1100元與1200元

Value function — concavity

PROBLEM 13:

(6,000, .25), or (4,000, .25; 2,000, .25).

$N = 68$ [18] [82]*

PROBLEM 13′:

(−6,000, .25), or (−4,000, .25; −2,000, .25).

$N = 64$ [70]* [30]

Applying equation 1 to the modal preference in these problems yields

$$\pi(.25)v(6{,}000) < \pi(.25)[v(4{,}000) + v(2{,}000)] \quad \text{and}$$
$$\pi(.25)v(-6{,}000) > \pi(.25)[v(-4{,}000) + v(-2{,}000)].$$

Hence, $v(6{,}000) < v(4{,}000) + v(2{,}000)$ and $v(-6{,}000) > v(-4{,}000) + v(-2{,}000)$. These preferences are in accord with the hypothesis that the value function is concave for gains and convex for losses.

Value function — steepness

- 損失1000元少的效用大於獲利1000元多的效用

generally increases with the size of the stake. That is, if $x > y \geq 0$, then $(y, .50; -y, .50)$ is preferred to $(x, .50; -x, .50)$. According to equation (1), therefore,

$$v(y)+v(-y)>v(x)+v(-x) \quad \text{and} \quad v(-y)-v(-x)>v(x)-v(y).$$

Setting $y=0$ yields $v(x)<-v(-x)$, and letting y approach x yields $v'(x)<v'(-x)$, provided v', the derivative of v, exists. Thus, the value function for losses is steeper than the value function for gains.

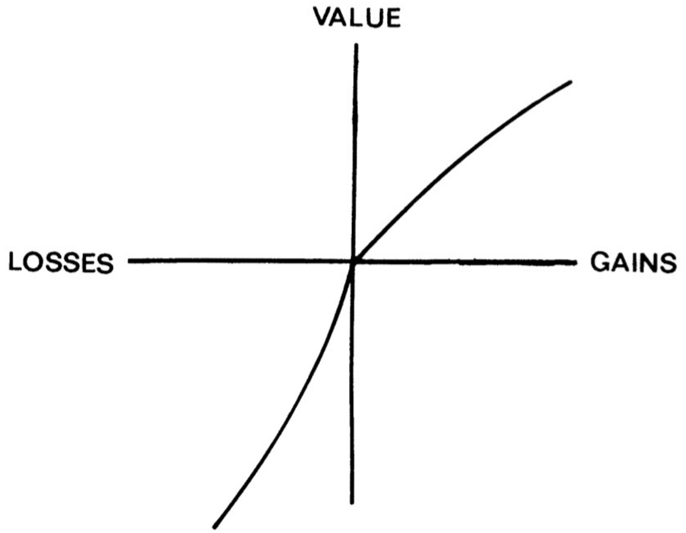


FIGURE 3.—A hypothetical value function.

The Weighting Function — extreme event

- 為什麼會買彩卷，對於極端事件的刻畫

Furthermore, we propose that very low probabilities are generally overweighted, that is, $\pi(p) > p$ for small p. Consider the following choice problems.

PROBLEM 14:

	(5,000, .001),	or	(5).
$N = 72$	[72]*		[28]

PROBLEM 14′:

	(−5,000, .001),	or	(−5).
$N = 72$	[17]		[83]*

Note that in Problem 14, people prefer what is in effect a lottery ticket over the expected value of that ticket. In Problem 14′, on the other hand, they prefer a small loss, which can be viewed as the payment of an insurance premium, over a small probability of a large loss. Similar observations have been reported by Markowitz [**29**]. In the present theory, the preference for the lottery in Problem 14 implies $\pi(.001)v(5{,}000) > v(5)$, hence $\pi(.001) > v(5)/v(5{,}000) > .001$, assuming the value function for gains is concave. The readiness to pay for insurance in Problem 14′ implies the same conclusion, assuming the value function for losses is convex.

The Weighting Function — subadditivity

- $\pi(0) = 0, \pi(1) = 1$
- 對於機率 $p + q = 1, \pi(p) + \pi(q) \leq 1$,for $0 < pq < 1$

$$v(2{,}400) > \pi(.66)v(2{,}400) + \pi(.33)v(2{,}500), \quad \text{i.e.,}$$

$$[1 - \pi(.66)]v(2{,}400) > \pi(.33)v(2{,}500) \quad \text{and}$$

$$\pi(.33)v(2{,}500) > \pi(.34)v(2{,}400); \quad \text{hence,}$$

$$1 - \pi(.66) > \pi(.34), \quad \text{or} \quad \pi(.66) + \pi(.34) < 1.$$

Applying the same analysis to Allais' original example yields $\pi(.89) + \pi(.11) < 1$,

The Weighting Function — monotone increase

Recall that the violations of the substitution axiom discussed earlier in this paper conform to the following rule: If (x, p) is equivalent to (y, pq) then (x, pr) is not preferred to (y, pqr), $0 < p, q, r \leqslant 1$. By equation (1),

$$\pi(p)v(x) = \pi(pq)v(y) \quad \text{implies} \quad \pi(pr)v(x) \leqslant \pi(pqr)v(y); \quad \text{hence,}$$

$$\frac{\pi(pq)}{\pi(p)} \leqslant \frac{\pi(pqr)}{\pi(pr)}.$$

Thus, for a fixed ratio of probabilities, the ratio of the corresponding decision weights is closer to unity when the probabilities are low than when they are high. This property of π, called subproportionality, imposes considerable constraints on the shape of π: it holds if and only if $\log \pi$ is a convex function of $\log p$.

It is of interest to note that subproportionality together with the overweighting of small probabilities imply that π is subadditive over that range. Formally, it can be shown that if $\pi(p) > p$ and subproportionality holds, then $\pi(rp) > r\pi(p)$, $0 < r < 1$, provided π is monotone and continuous over $(0, 1)$.

From original to cumulative prospect theory

原本的論文只確定大概，對於極端事件，或到底general form並沒有刻畫函數或模型，要等到1992年康納曼和特沃斯基再次提出新的理論cumulative prospect theory

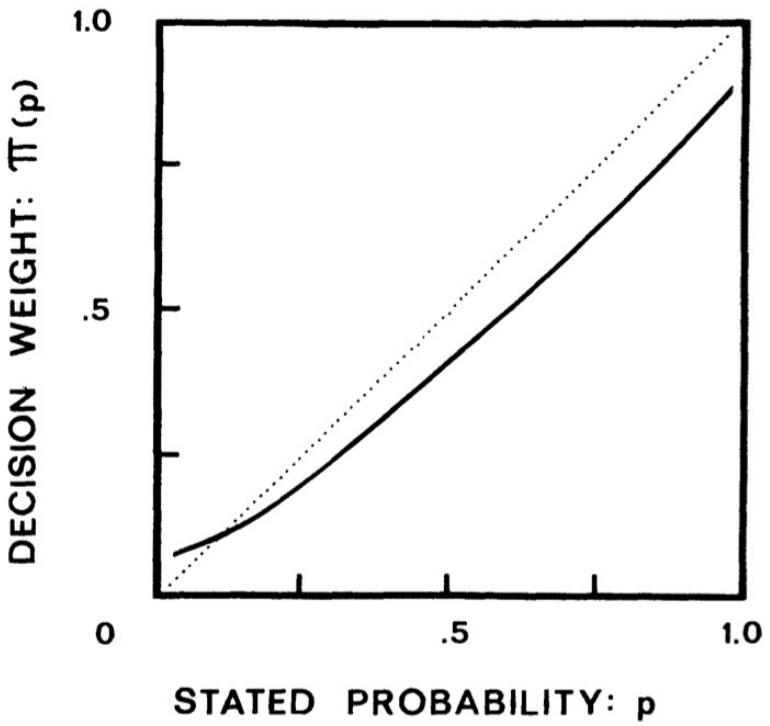
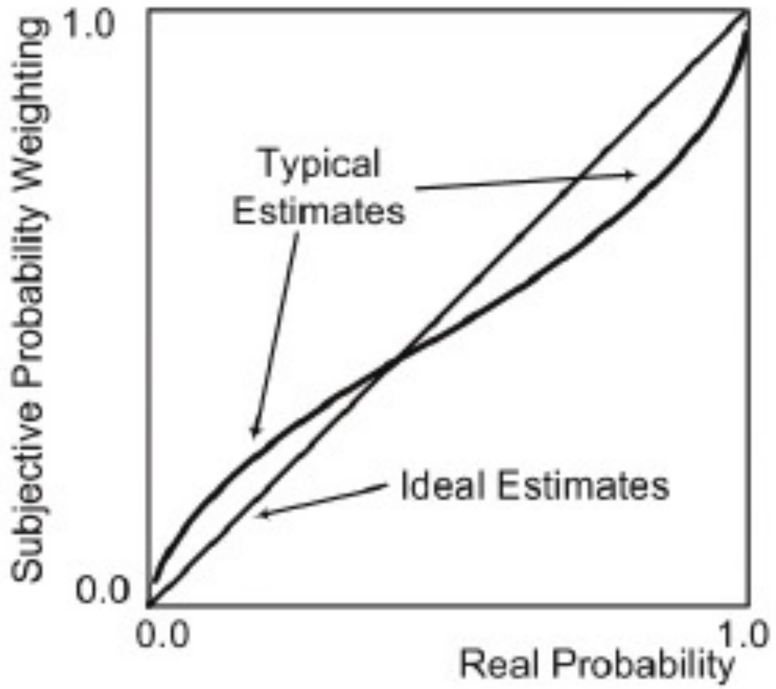


FIGURE 4.—A hypothetical weighting function.



Shifts of reference

- 想像一個剛損失2000元的商人，面對一個選擇(2,000,0.5) 和 (1,000).
- he is likely to code the problem as a choice between

(-2,000, 0.5) and (-1,000) rather than as

a choice between (2,000,0.5) and (1,000).

如我們所見，前者比後者引起了更多的風險愛好。

Q&A

Thanks for listening!